Lecture 1 – Nanoscale MOSFETs

- Course Structure
- Basic Concepts (1-38)

Course Layout

- 7.5 ECTS

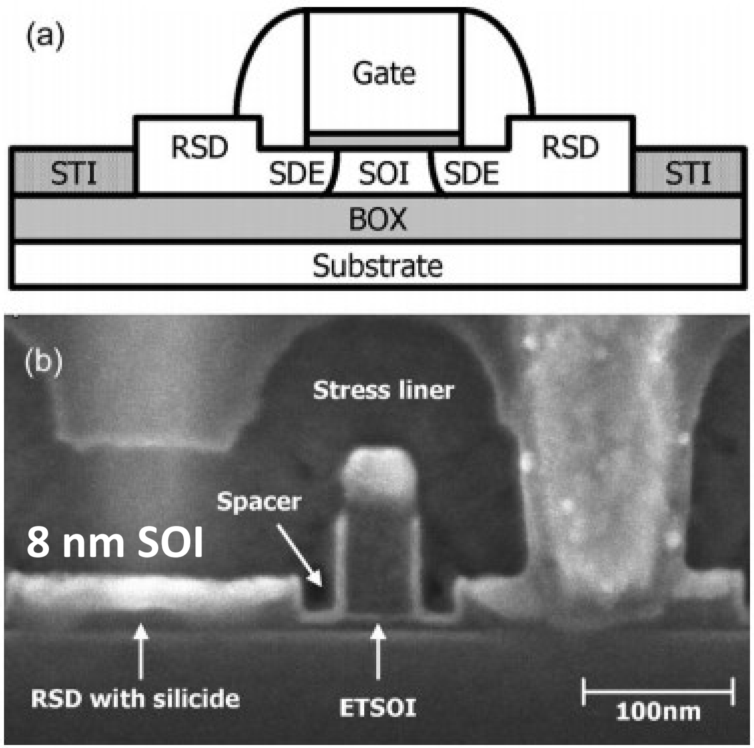
- 7 Lectures
- 7 Excercises

- Written exam (t.b.d)

Textbook:
Nanoscale Transistors: Device Physics, Modeling and Simulation
by Lundstrom/Guo. *Avaliable electronically*.
We will cover everything except the last chapter(1-181).

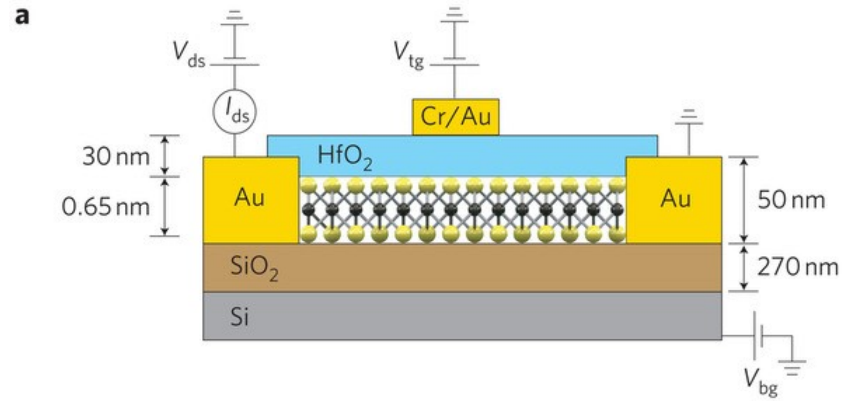
Motivation I

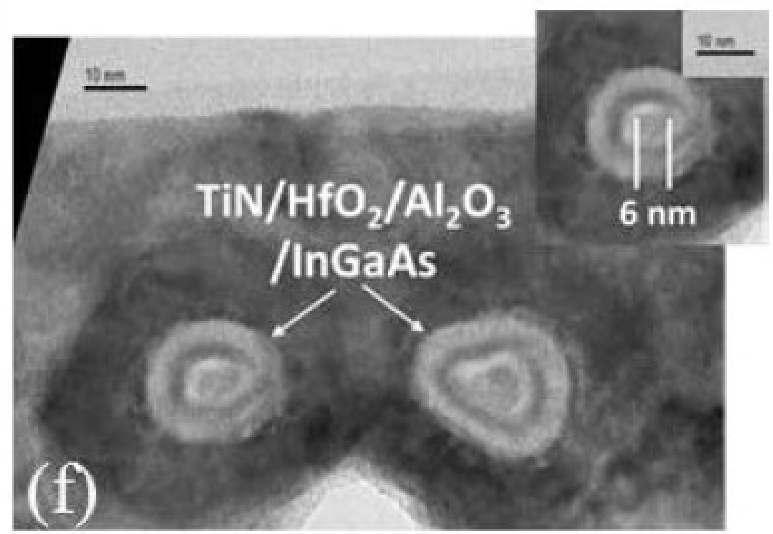
Modern transistors have *thin* 1D or 2D bodies



Undoped-Body Extremely Thin SOI MOSFETs With Back Gates

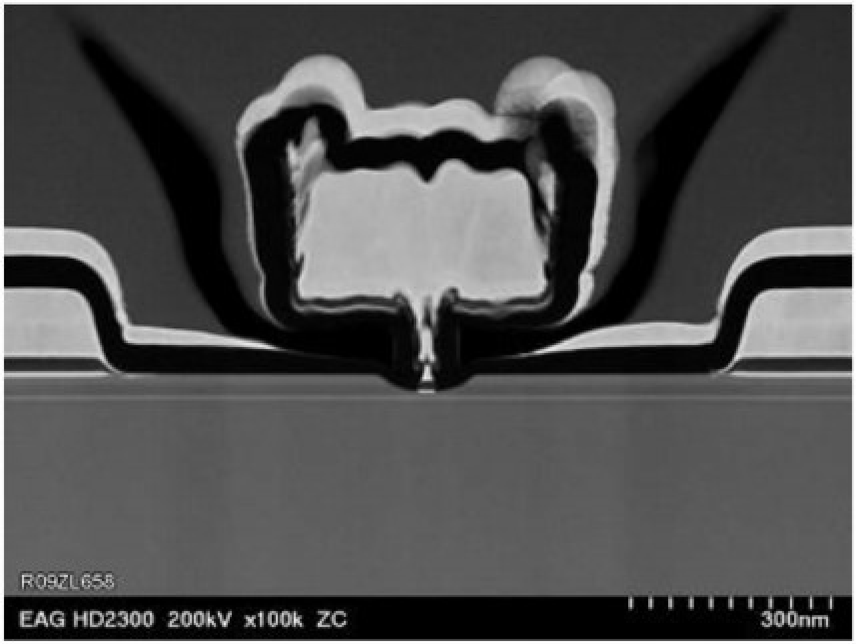
Amlan Majumdar, Zhibin Ren, Steven J. Koester, *Senior Member, IEEE*, and Wilfried Haensch, *Senior Member, IEEE*





Motivation II

Modern transistors have very short gate lengths



A STEM image of a 30 nm InP HEMT.

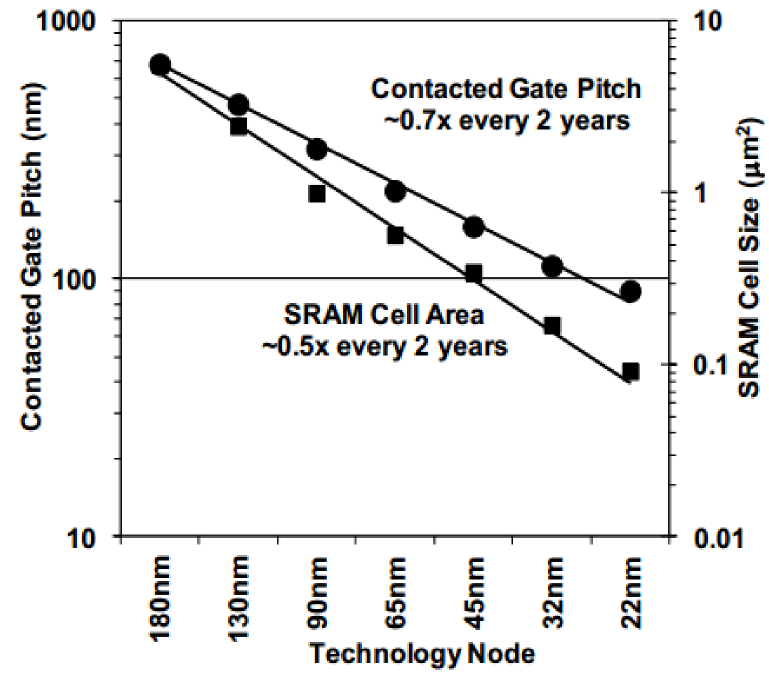


Fig. 4. Contacted gate pitch and SRAM cell size scaling trend for Intel's technology nodes.

$L_g < \lambda$ – diffusive transport gives a poor description of the device performance

HEMTs: 10-30 nm Lg. 14 nm Si FINFET: 20 nm Lg.

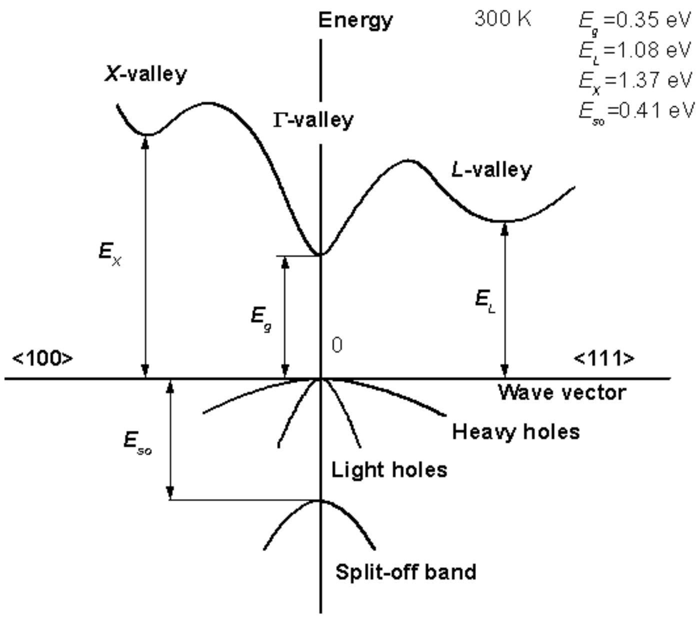
We need to understand the behaviour of close to ballistic FETs

Course Topics

Contemporary description of MOSFETs

- Basic semiconductor concepts (1)
- Basic MOSFET / CMOS physics (2)
- 2D Ballistic FETs (3-4)
- Scattering (5)
- 1D Nanowire and CNT FETs (6)
- ~~Molecular FETs – Single Electron Transistors (7)~~
- 1D Tunneling Field Effect Transistors. Ferroelectric FETs. (7).

Band Structure, Group Velocity



- Direct band gap semiconductors
- Isotropic Γ-valley
- Parabolic bands for small *E*

$$E(\boldsymbol{k}) = E_C + \frac{\hbar^2}{2m^*}\left(k_x^2 + k_y^2 + k_z^2\right)$$

- Indirect band gap semiconductors (Si, Ge)
- Multiple *anisotropic X*/*L*-valleys
- Parabolic bands for small *E*

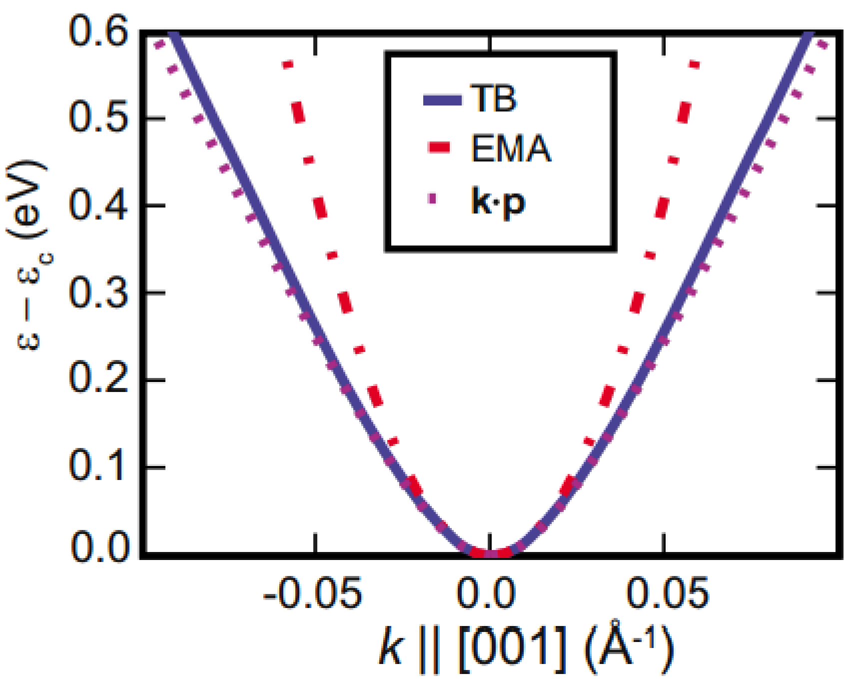
$$E(\boldsymbol{k}) = E_C + \frac{\hbar^2}{2}\left(\frac{k_x^2 + k_y^2}{m_t} + \frac{k_z^2}{m_l}\right)$$

The group velocity for a conduction band electron

$$v_x = \frac{1}{\hbar}\frac{dE}{dk_X}$$

Nonparabolicity

InAs



E >> E_c introduces nonparabolicity

$$E(k)\left(1+\alpha E(k)\right)=\frac{\hbar^2}{2m^*}\left(k_x^2+k_y^2+k_z^2\right)$$

(2-band kp)

α: 2 eV-1 (InAs)
0.5 eV-1 (Si)

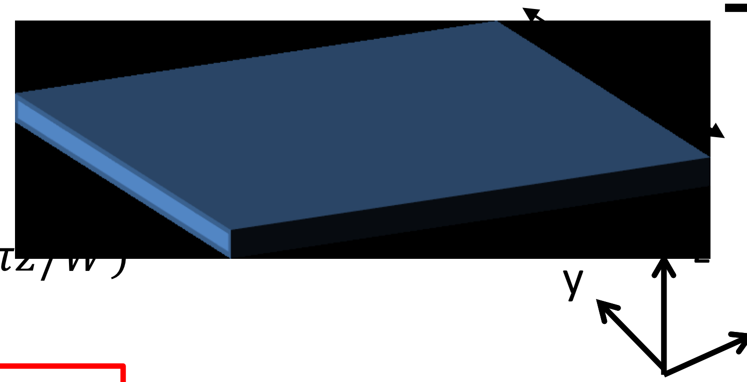
TB: Atomistic bandstructure modeling
EMA: Effective mass
kp: 2-band k dot p

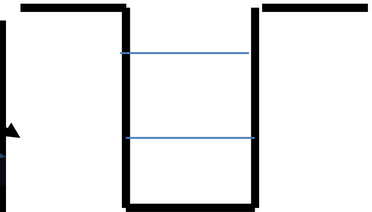
2D / 1D structures

The channel of a modern FET is 2D today - 1D tomorrow (?)

$$\psi(r) = \phi(z)\frac{1}{\sqrt{A}}e^{ik_x x + ik_y y}$$

$$\phi(z) = \sqrt{\frac{2}{W}}\sin(k_n z) = \sqrt{\frac{2}{W}}\sin(n\pi z/W)$$

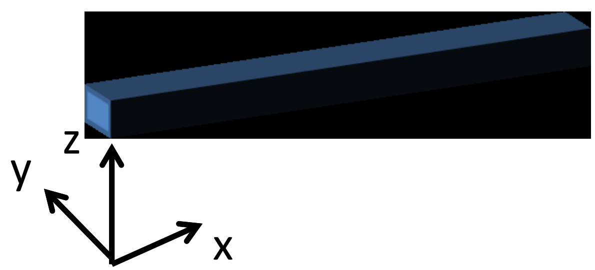




$$\boxed{\varepsilon_n = \frac{\hbar^2 n^2 \pi^2}{2m^* W^2} \qquad E(\boldsymbol{k}) = \varepsilon_n + \frac{\hbar^2 k_\parallel^2}{2m^*}}$$

$$\psi(r) = \phi(y,z)\frac{1}{\sqrt{L}}e^{ik_x x}$$

$$\phi(y,z) \propto \sin\left(\frac{n\pi z}{W_z}\right)\sin\left(\frac{n\pi y}{W_y}\right)$$



$$\boxed{\begin{aligned} \varepsilon_{n,m} &= \frac{\hbar^2 \pi^2}{2m^*}\left(\frac{n^2}{W_z^2} + \frac{m^2}{W_y^2}\right) \\ E(k) &= \varepsilon_{n,m} + \frac{\hbar^2 k_x^2}{2m^*} \end{aligned}}$$

Fermi-Dirac Integrals

$$F_j(\eta_F) = \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{\xi^j}{1 + e^{\xi - \eta_F}} d\xi$$

$$\frac{dF_j}{d\eta_F} = F_{j-1}$$

$$F_0(\eta_F) = \log(1 + e^{\eta_F})$$

$$\eta_F = \frac{E_F - E_n}{kT}$$

In the non-degenerate limit: $\eta_F \ll 0$

$$F_j(\eta_F) \approx e^{\eta_F}$$

In the degenerate limit: $\eta_F \gg 0$

$$F_j(\eta_F) \approx \eta_F^{j+1} / \Gamma(j+2)$$

Gamma function

$$\Gamma(0.5) = \sqrt{\pi}$$

$$\Gamma(1) = 1$$

$$\Gamma(1.5) = \frac{\sqrt{\pi}}{2}$$

$$\Gamma(2) = 1$$

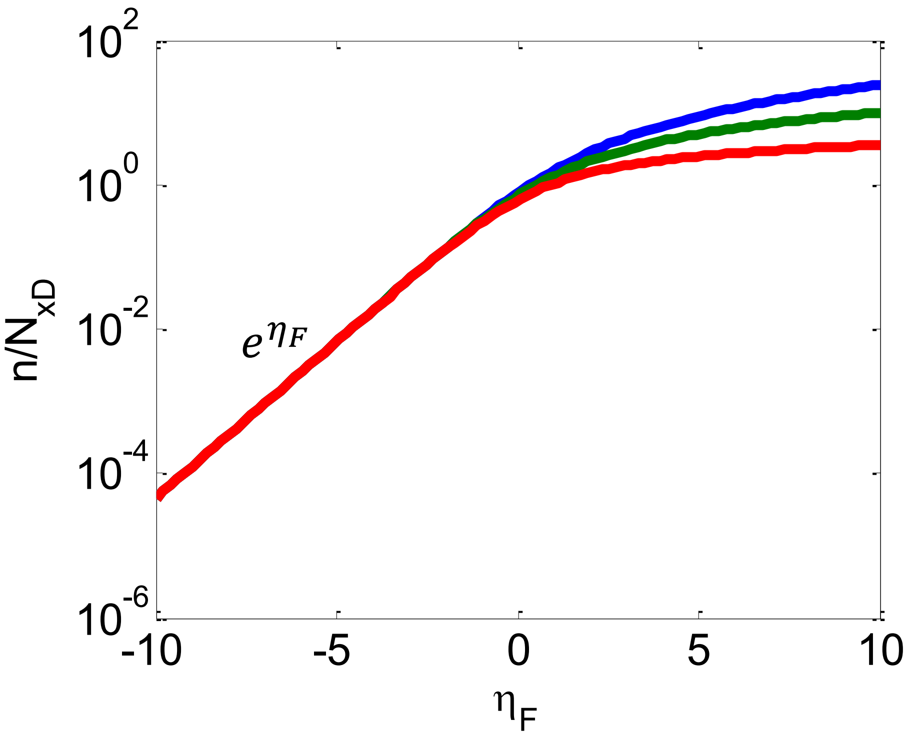
$$\Gamma(2.5) = \frac{3\sqrt{\pi}}{4}$$

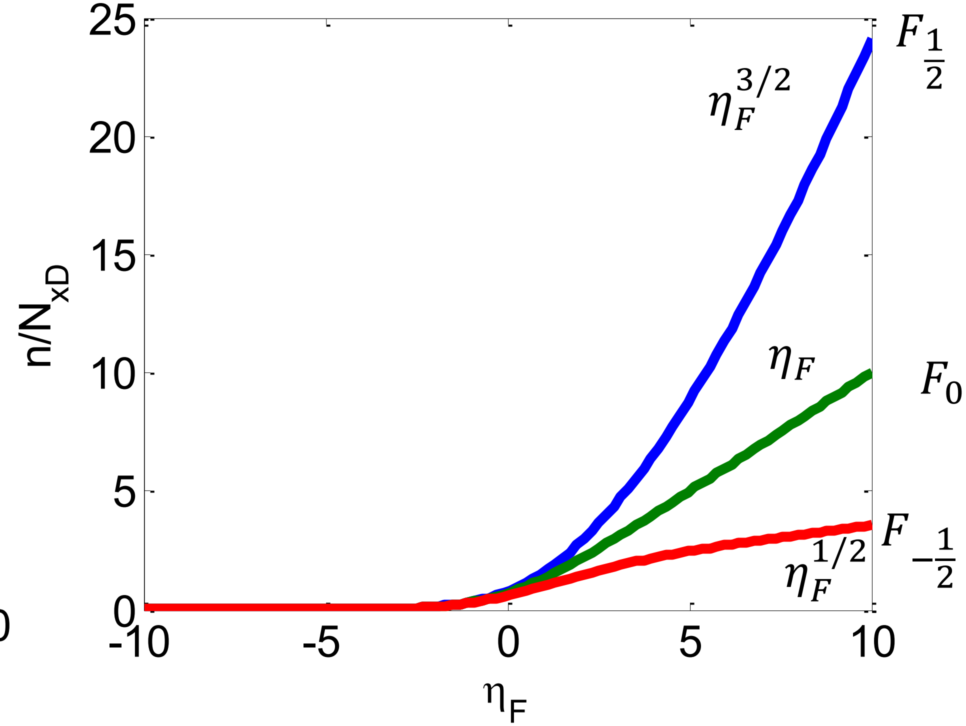
n positive integer $\quad \Gamma(n) = (n-1)!$

$$\Gamma(p+1) = p\Gamma(p)$$

Notes on FD/integrals:
http://arxiv.org/ftp/arxiv/papers/0811/0811.0116.pdf

Fermi-Dirac Integrals





In the non-degenerate limit:

$$F_j(\eta_F) \approx e^{\eta_F}$$

In the degenerate limit:

$$F_j(\eta_F) \approx \eta_F^{j+1} / \Gamma(j+2)$$

$$\eta_F = \frac{E_F - E_n}{kT}$$

$$\Gamma(0.5+2) = \frac{3\sqrt{\pi}}{4}$$

$$\Gamma(0+2) = 1$$

$$\Gamma(-0.5+2) = \frac{\sqrt{\pi}}{2}$$

Carrier Statistics: 3D,2D,1D

Density of states:

$$D_{3D} = \frac{(2m^*)^{\frac{3}{2}}}{2\pi^2\hbar^3}\sqrt{E - E_C}$$

$$D_{2D} = \frac{m^*}{\pi\hbar}$$

$$D_{1D} = \frac{\sqrt{2m^*}}{\pi\hbar}\frac{1}{\sqrt{E - E_C}}$$

Effective density of states:

$$N_{3D} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{\frac{3}{2}}$$

$$N_{2D} = \frac{m^* kT}{\pi\hbar^2}$$

$$N_{1D} = \frac{\sqrt{\frac{2m^* kT}{\pi}}}{\hbar}$$

$$v_x = \frac{1}{\hbar}\frac{dE}{dk_X}$$

We will derive these later on.

$$n_0(k_F) = \sum_k f_0(k, k_F) = \int_0^\infty f_0(E, E_F) D(E) dE$$

$$F_j(\eta_F) = \frac{1}{\Gamma(j+1)}\int_0^\infty \frac{\xi^j d\xi}{1 + e^{\xi - \eta_F}}$$

$$\eta_F = (E_F - E_C)/kT$$

$n_0 = N_{3D} F_{1/2}(\eta_F)$ 3D

$n_s = N_{2D} F_0(\eta_F)$ 2D

$n_L = N_{1D} F_{-1/2}(\eta_F)$ 1D

Semi-classical transport

$$\frac{\partial f(x,p,t)}{\partial t} + v_x \frac{\partial f}{\partial x} - qE_x \frac{\partial f}{\partial p_x} = \hat{C}(x,p,t)f$$

Boltzmann Transport Equation

f: distribution function. C: collision operation

Zeroth Moment: continuity equation
First Moment: drift diffusion
2nd Moment: energy balance

$$J_{nx} = qn\mu_n E_x + \frac{2}{3}\mu_n \frac{dW}{dx}$$

$$\mu_n = \frac{q\tau_m}{m^*}$$

$$W \approx \frac{3}{2}nkT$$

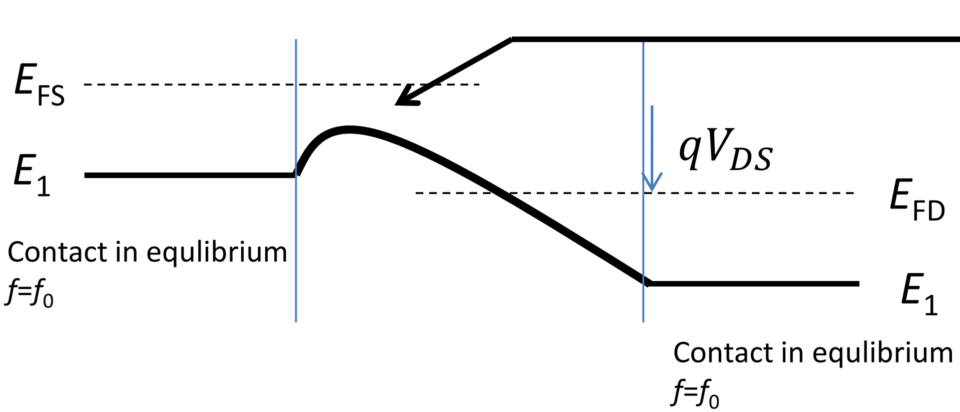
Kinetic energy near equilibrium

High field, short devices:

$$\left.\begin{array}{ll} \tau_m(E) & \text{Scattering increases with energy} \\ W > \frac{3}{2}nkT & \text{Mean energy larger} \end{array}\right\}$$

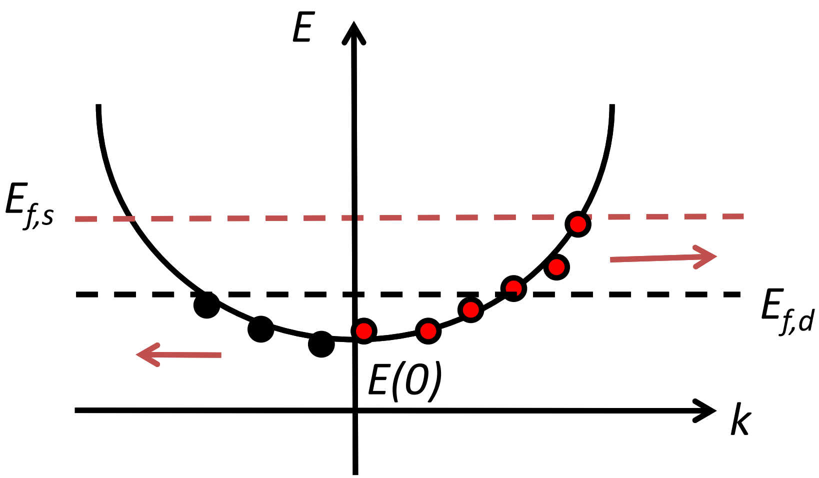
Drift-diffusion is a poor approximation for nm-scale transistors!

Ballistic Transport – Directed moments



$$v_x \frac{\partial f_0}{\partial x} - qE_x \frac{\partial f_0}{\partial p_x} = 0$$

f=f_0 in the active region – but with two Fermi levels!



$$n_s^+ = \frac{1}{A} \sum_{\boldsymbol{k_x, k_y > 0}} f_0\left(E_{F,d}, E(k)\right)$$

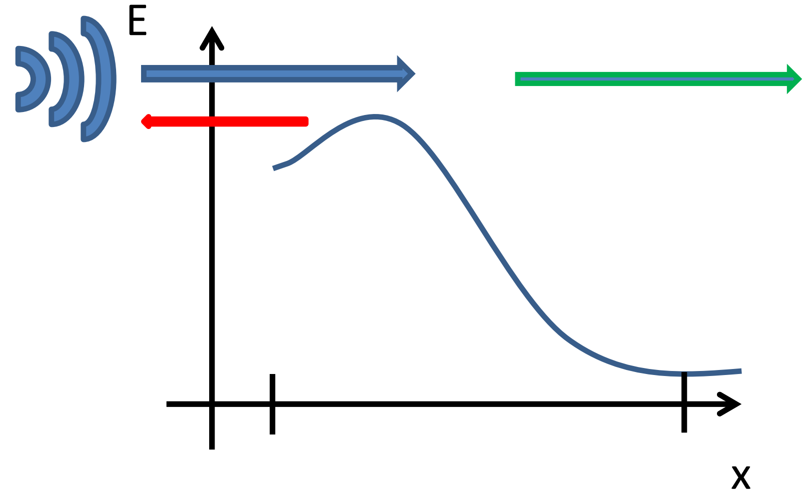
$$J^+ = \frac{1}{A} \sum_{k_x, k_y > 0} q v_x f_0\big(E(k) - E_{F,d}\big)$$

$$\sum_{\boldsymbol{k}} g(\boldsymbol{k}) \rightarrow \frac{L^d}{(2\pi)^d} \int_{\boldsymbol{k}} g(\boldsymbol{k}) d\boldsymbol{k}$$

We will work out these expressions in detail for the 1D and 2D FETS in comming lectures

Quantum Transport – 1D

The Schrödinger Equation with open boundary conditions:



$$\left.\begin{array}{ll} \psi(x) = 1e^{ik_1x} + re^{-ik_1x} & x<0 \\ \psi(x) = te^{-ik_2x} & x>L \end{array}\right\} \psi(x,k)$$

Local Density of states / electron density:

$$LDOS_1(x,E) \equiv \left[\frac{1}{\pi}\frac{dk_1}{dE}\ |\psi(x,k_1)|\right]$$

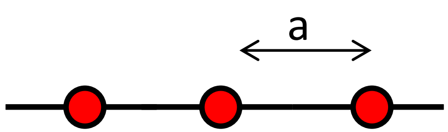
$$n_1(x,E) = f_0(E - E_F)LDOS_1(x,E)$$

Current:

$$I_D(E) = \frac{2q}{h}\ T_{1-2}(E)[f_0(E - E_F) - f_0(E - E_F - qV_D)]$$

$$T_{1-2}(E) = 1 - |\psi(0,E) - 1|^2$$

Quantum Transport – Numerical Solutions



$$\frac{d^2}{dx^2}f(x_n) \rightarrow \frac{\left(-f(x_{n-1}) + 2f(x_n) - f(x_{n+1})\right)}{a^2}$$

$[E\boldsymbol{I} - \boldsymbol{H} - \boldsymbol{\Sigma_1} - \boldsymbol{\Sigma_2}]\psi = i\boldsymbol{\gamma_1}$ N×N matrix

Self-Energies – models the open boundary contacts:

$\Sigma_1(i,j) = -t_0 e^{ik_1 a}\delta_{1,i}\delta_{1,j}$

$\Sigma_2(i,j) = -t_0 e^{ik_1 a}\delta_{N,i}\delta_{N,j}$

N×N matrices with 1 element ≠0

$$t_0 = \frac{\hbar^2}{2m^* a^2}$$

Source Injection:

$\gamma_1 = i[\Sigma_1(1,1) - \Sigma_1^*(1,1)] = \hbar\dfrac{v(k)}{a}$ N×1 vector with (1,1) element ≠0

$\psi = i\boldsymbol{G}\boldsymbol{\gamma}$

$\boldsymbol{G} = [E\boldsymbol{I} - \boldsymbol{H} - \boldsymbol{\Sigma_1} - \boldsymbol{\Sigma_2}]^{-1}$ Retarded Green's function

$\psi_{x_n}(E)$

Quantum Transport – NEFG

$$\boldsymbol{G_r} = [E\boldsymbol{I} - \boldsymbol{H} - \boldsymbol{\Sigma_1} - \boldsymbol{\Sigma_2}]^{-1}$$

$A_1(E) = \boldsymbol{G}\Gamma_1\boldsymbol{G}^\dagger$ *Spectral Density ≈ LDOS*

$$\Gamma_1 = i(\Sigma_1 - \Sigma_1^\dagger)$$

$G_1^n(E) = f_0(E_{F1} - E)\dfrac{A_1}{2\pi}$ *Correlation Function*

$T_{1-2}(E) = trace(\Gamma_1 G \Gamma_2 G^\dagger)$ *Transmission*

$$I_D(E) = \frac{2q}{h} T_{1-2}(E)[f_0(E - E_F) - f_0(E - E_F - qV_D)]$$

If **H** is a real space representation:

Tr(A_1) equals to the local density of states

Tr($G_1{}^n$) equals to the position dependent electron concentraion

Can treat ***scattering*** and ***many-body*** effects.

$$\boldsymbol{G} = [E\boldsymbol{I} - \boldsymbol{H} - \boldsymbol{\Sigma_1} - \boldsymbol{\Sigma_2} - \boldsymbol{\Sigma_s}]^{-1}$$

Self-Energy due to scattering

Excercises

Download and work on the excercise sets – 1 every week.